Optical probing of electronic fractional quantum Hall states

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We report on the observation of a fine structure in the photoluminescence emission of high-mobility GaAs/ AlGaAs single heterojunctions in the fractional quantum Hall regime. A splitting of the emission band into three lines is found both at filling factor $\nu = 2/3$ and in the region $2/5 > \nu > 1/3$. The dependencies on filling factor, electron density, and temperature show that the fine structure arises from the recombination of fractionally charged elementary excitations of the two-dimensional electron liquid and an itinerant valence-band hole. These quasiparticle excitations (anyon excitons) exhibit a dispersion relation with an absolute minimum at large momentum, leading to a characteristic, broad emission band at the low-energy side of the photoluminescence spectrum around $\nu = 1/3$.

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Electron-electron (e-e) interactions in a two-dimensional electron gas (2DEG) under a strong magnetic field (B) give rise to highly correlated electron states. These states form the incompressible quantum liquid (IQL) (Refs. [1](#page-3-1) and [2](#page-3-2)) under-lying the fractional quantum Hall (FQH) effect.^{3,[4](#page-3-4)} The elementary excitations of the IQL are quasielectrons (QEs) and quasiholes (QHs) with a fractional charge and a finite exci-tation energy.^{1,[2](#page-3-2)[,4](#page-3-4)} The existence of fractionally charged quasiparticles has been demonstrated first by magnetotransport experiments^{3[,4](#page-3-4)} and later on by shot noise⁵ and cyclotron resonance⁶ measurements. Anomalies in the photoluminescence (PL) of a 2DEG with photoexcited holes in the FQH regime also evidence correlated electron states $7-12$ $7-12$ although the physical nature of these anomalies is still not fully understood.

The origin of the PL in the FQH regime of 2D systems, such as quantum wells (QWs) and heterojunctions (HJs), depends strongly on the relative strength of the *e*-*e* and electron-hole $(e-h)$ interactions.^{13–[18](#page-3-10)} In most QW systems studied, the distance *d* between the electron and hole confinement layers is small, and the strength of the *e*-*e* and *e*-*h* interactions is comparable. In this case the electron correlations are strongly affected by the photoinjected valence-band holes, and the PL emission arises from neutral and charged excitons, rather than from the correlated 2DEG itself.^{11[,18](#page-3-10)} This has led to a wide variety of PL features: at low electron densities $(n_e < 10^{11} \text{ cm}^{-3})$ the distinct PL peaks arise from neutral and charged excitons that can be regarded as isolated from the 2DEG.¹¹ At higher n_e , the (charged) excitons interact with fractionally charged 2DEG excitations. In most cases this leads to *doublets* in the PL spectra and to *discontinuities* in the PL peak energy at fractional filling factors ν $= 2 \pi n_e l_B^2$ (Refs. [7](#page-3-7) and [12](#page-3-8)) $(l_B = \sqrt{\hbar / eB})$ is the magnetic length, e the electron charge, and \hbar Planck's constant).

In this Rapid Communication we study a different type of 2D system: single GaAs/AlGaAs HJs in which the *e*-*e* correlations are probed optically by radiative recombination of the 2DEG with itinerant valence holes (2De-h PL). The internal electric field in the HJ results in a large *e*-*h* separation

and, therefore, a much weaker *e*-*h* interaction. 2D*e*-*h* PL measurements in HJs in the FQH regime are scarce, and limited to the detection of PL anomalies at integer filling factors $8,19-22$ $8,19-22$ $8,19-22$ and to the observation of a doublet structure at $\nu = 2/3$.⁸ Theoretically, a multiple PL line structure (with more than two peaks) is predicted when $d > l_B$ arising from the radiative recombination of a photoexcited valence hole with several fractionally charged excitations of the 2DEG[.14](#page-3-15)[–16](#page-3-16)

We report on the observation of a *triplet* fine structure in the PL spectra around both $\nu = 2/3$ and $\nu = 1/3$. With increasing magnetic field (at $2/5 > \nu > 1/3$) a weak, broad PL line emerges at low photon energy, and it increases in intensity with decreasing temperature and increasing magnetic field (increasing d/l_B ratio). We attribute this new line to indirect recombination of an anyon exciton whose dispersion exhibits an absolute "magnetoroton" minimum at finite momentum $k \sim 1/l_B$ for large *e*-*h* separation $(d/l_B > 1).$ ^{[13](#page-3-9)}

We have studied several high-mobility $(\mu > 5 \times 10^6 \text{ cm}^2/V \text{ s})$ GaAs/AlGaAs single HJs having electron densities in the range of $n_e = (1.4 - 2.7) \times 10^{11}$ cm⁻². The samples were grown by molecular-beam epitaxy along the (100) direction and have a thick GaAs buffer layer (width of 1 μ m). The *n*-doped δ layer is separated from the interface by a wide (>80 nm) undoped AlGaAs layer. Polarized PL spectra were measured at temperatures T_L =0.4 and 1.2 K in magnetic fields *B* up to 33 T in Faraday configuration. The sample was illuminated by a Ti:Sapphire laser tuned below the AlGaAs barrier with low power density (≤ 5 mW/cm²). Left (σ^-) and right (σ^+) circularly polarized PL was collected using optical fibers and circular polarizers. The spectra were dispersed by a single grating spectrometer and recorded with a liquid-nitrogen cooled charge-coupled-device camera (0.1) meV resolution).

The HJs studied all show a similar PL evolution with ν . Figure [1](#page-1-0) displays the polarized PL of a HJ with $n_e = 2.7 \times 10^{11}$ cm⁻² in magnetic fields *B*=0-32 T. At *B*=0 only PL from bulk excitons in the wide GaAs layer is observed around 1.515 eV.²³ With increasing *B*, the energy of the bulk excitons shows a diamagnetic shift and the intensity

FIG. 1. (Color online) Contour plot of the polarized emission spectra of a single HJ $(n_e=2.7\times10^{11} \text{ cm}^{-2})$ at $T_L=0.4 \text{ K}$ in magnetic fields up to 32 T. The false color scale indicates the intensity of the emission.

in σ^+ polarization increases, while the intensity in σ^- polarization decreases. We have observed similar behavior in a pure, bulk GaAs reference sample.

At $\nu = 2$, the excitonic emission abruptly transfers its intensity to the radiative recombination of the 2DEG with valence holes (2De-*h* PL) in the σ^- polarization, and the PL peak energy jumps down to the energy of the optical transition between the lowest Landau levels of the 2D electrons and valence holes. $19-22$ $19-22$ This abrupt transition from bulk exciton to 2D*e*-*h* PL was phenomenologically explained by the dissociation of free excitons into electrons and itinerant valence holes near the 2DEG at integer ν , especially at $\nu < 2^{21,22}$ $\nu < 2^{21,22}$ $\nu < 2^{21,22}$

At $\nu = 1$, the 2De-*h* PL peak energy exhibits a sharp discontinuity.^{19[,22](#page-3-14)[,24](#page-3-19)} Experiments with HJs of various densities show that the interplay between hole repulsion (due to the built-in HJ electric field) and hole attraction to the 2DEG that depends on the 2DEG screening properties in magnetic field) controls the exciton to 2De-h PL changeover at integer . Here, we use the sharp discontinuity of the 2D*e*-*h* PL energy at $\nu = 1$ to determine the 2DEG density, which closely equals that obtained from transport measurements.

The behavior at integer ν points out that the 2De-*h* emission in our high-quality HJs originates from 2DEG electrons and *itinerant valence holes*, without any noticeable effect of disorder.^{21,[22](#page-3-14)} In the following we will focus on the PL features appearing at $\nu < 1$. In the range $1 > \nu > 0.68$, the PL spectrum consists of a single narrow line $(0.2 \text{ meV line}$ width) whose energy shifts nearly linearly with increasing *B*. At fractional filling factors $\nu = 2/3$ and $2/5 > \nu > 1/3$ a fine structure emerges in the PL spectrum. Figure $2(a)$ $2(a)$ shows the PL spectra of a HJ sample with $n_e = 2.2 \times 10^{11}$ cm⁻². Around $\nu = 2/3$ ($B = 13.7$ T), the narrow PL line abruptly splits into three lines, with intensity transferring to the lowest-energy line. At higher *B*, the three PL lines merge gradually into a single line which is slightly broader than 0.2 meV. The energy and width of the lines in the vicinity of $\nu = 2/3$ are obtained by fitting the PL spectra with three Lorentzians as shown in Fig. $2(b)$ $2(b)$. We find that the three peaks are nearly

FIG. 2. PL of a 2DEG $(n_e=2.2\times10^{11} \text{ cm}^{-2})$ around $\nu=2/3$ at 0.4 K in σ^- polarization. (a) The spectra for $B = 13.3 - 14.3$ T measured with 0.02 T step. (b) PL spectra fitted with three Lorentzian peaks.

equidistant with an energy separation of 0.15 meV and a width of 0.2 meV. Note that this observation of three peaks at ν = 2/3 differs from previous PL experiments on a similar HJ sample, $\frac{8}{3}$ where only two peaks were resolved.

At higher *B* corresponding to $\nu < 1/2$, a different spectral structure emerges in the PL spectra, displayed in Fig. $3(a)$ $3(a)$ for two HJs with $n_e = 2.7 \times 10^{11}$ cm⁻² (left panel) and n_e =1.9 × 10¹¹ cm⁻² (right panel). Figure [3](#page-2-0)(b) shows the PL peak positions for the former HJ with a linear energy subtracted $(\approx 0.79 \text{ meV/T})$ in order to highlight the energy splittings. As *B* increases from $\nu = 2/5$, first the intensity of the 2De-h PL peak [full circles in Fig. [3](#page-2-0)(b)] decreases and is transferred to a new low energy, rather weak and broad PL band (triangles). When B is increased further, the main 2D*e*-*h* PL peak splits into a doublet with a maximal splitting Δ_2 = 0.4 meV at 30.5 T. This doublet gains intensity from the lowest PL peak which is located at approximately $\Delta_1 = 1.2$ meV below the doublet center. At $\nu \leq 1/3$, the doublet structure of the peak abruptly disappears, together with the low-energy PL band. We note that in some HJ samples the electron density slightly decreases at high B (below ν = 1/2).^{[25](#page-3-20)} The PL anomalies corresponding to specific filling factors (e.g., $\nu = 2/5$ and $1/3$) are, therefore, observed at lower *B* values than expected from the nominal electron density.

The emission of the bulk GaAs excitons [the highest photon energy peak in Fig. $3(b)$ $3(b)$, squares] does not show any anomalous behavior, which strongly suggests that the observed features in the 2D*e*-*h* PL are evidence of FQH states. This is supported by the fact that we only observe the PL structure at low temperatures (0.4 K). At T_L =1.2 K [cf. Fig. $4(b)$ $4(b)$, the 2De-*h* PL does not show any anomalies and consists of a single peak. Most importantly, the broad PL band at low energy increases in intensity with decreasing T_L , opposite to the behavior reported before for PL doublets in QWs[.7](#page-3-7)[,12](#page-3-8)

Figure $4(a)$ $4(a)$ shows PL spectra of three HJs containing 2DEGs with different densities taken in the region between $\nu = 2/5$ and $\nu = 1/3$ where the observed doublet splitting is maximal. The dashed lines are a guide to the eye to indicate

FIG. 3. (a) PL of two HJs $(n_e=2.7\times10^{11} \text{ cm}^{-2}$ and $n_e = 1.9 \times 10^{11}$ cm⁻²) between $\nu = 2/5$ and $\nu = 1/3$ at $T_L = 0.4$ K. The dashed lines are a guide to the eye to indicate the new spectral features at fractional filling factors. (b) Energy difference between the transitions with respect to the center of the doublet peak (i.e., a linear energy subtracted) for the HJ with $n_e = 2.7 \times 10^{11}$ cm⁻². The size of the symbols represents the amplitude of the transitions. For visibility the symbols are rescaled to the maximum of each transition.

the splittings. The splitting increases with increasing electron density and with the magnetic field. The inset of Fig. $4(a)$ $4(a)$ shows the peak energies as function of $1/l_B$. Within the experimental accuracy, both the doublet splitting and the splitting between the doublet and the additional peak are inversely proportional to the magnetic length.

Given the fact that we observed the PL fine structure in all our high-mobility HJs we identify it as the intrinsic optical response of a 2DEG in the FQH regime, probed by a photoexcited hole at a distance larger than the magnetic length $(d > l_B)$. This fine structure consists of triplets, as opposed to a doublet PL structure observed previously[,7,](#page-3-7)[8](#page-3-12)[,12](#page-3-8) and which were the subject of numerous theoretical investigations, $13-18$ mostly performed around $\nu = 1/3$.

Apalkov and Rashba¹³ have first explained the doublet PL structure by identifying two recombination channels of anyon excitons: a direct recombination at $k \approx 0$ and an indirect (shake-up) recombination from the magnetoroton minimum at $k \sim 1/l_B$ that develops due to a modification of the anyon-exciton dispersion by the IQL. This picture seems to be an appropriate explanation for the two far-separated bands visible in our PL spectra at $2/5 > \nu > 1/3$. We assign the low-energy PL band to the indirect magnetoplasmon-assisted transitions that arise from an extensive area of *k* space (near $k \sim 1/l_B$) leading to *a relatively weak and broad PL line* having an energy below the *k*= 0 exciton PL peak. The

FIG. 4. (a) Spectra in the region between $\nu = 2/5$ and $\nu = 1/3$ at 0.4 K for three samples with electron densities $n_e = (1.4 - 2.7) \times 10^{11}$ cm⁻². The dashed lines are a guide to the eye to indicate the peak positions. Inset: doublet splitting (circles) Δ_2 and energy difference between the doublet and the additional peak (squares) Δ_1 as function of $1/l_B$ for the three different samples. The lines are a linear fit to the data with a slope of 8.1 ± 0.6 meV nm and 2.0 ± 0.2 meV nm, respectively. (b) Spectra at 0.4 and 1.2 K for the HJ sample with $n_e = 2.7 \times 10^{11}$ cm⁻².

energy splitting Δ_1 between these lines scales with the inverse magnetic length [Fig. $4(a)$ $4(a)$], as expected since the Coulomb energy in the FQH regime is proportional with e^2/l_B . The low-energy line gains intensity with decreasing temperatures, opposite to the temperature dependence of the doublets in the PL of QWs.^{7,[12](#page-3-8)} Our experimental findings are therefore consistent with the theoretical prediction that for $d > l_B$ the anyon-exciton dispersion develops an *absolute* magnetoroton minimum at finite momentum $k \sim 1/l_B$. Then, the competition of the direct and indirect (shake-up) channels gives rise to a spectrum consisting of bright, narrow $(k \approx 0)$ and weak, broad $(k \sim 1/l_B)$ PL lines with the latter becoming broader and more intense with increasing d/l_B or *lowering* T_L .^{[13](#page-3-9)}

Let us now discuss the second important feature of the PL fine structure at fractional ν , namely, the splitting of the main PL line into a doublet at $\nu = 1/3$ $\nu = 1/3$ (around 30 T in Fig. 3) and a triplet at $\nu = 2/3$ $\nu = 2/3$ $\nu = 2/3$ (around 13.6 T in Fig. 2). We associate those splittings into two or three *narrow* PL lines with the multiple-branch structure of the anyon exciton at $k=0$ that is due to internal degrees of freedom of QEs constituting the anyon exciton.¹⁴

Alternatively, MacDonald *et al.*[15](#page-3-21) and Chen *et al.*[16](#page-3-16) suggested that a multiple PL spectrum in the FQH regime can arise from recombination of valence holes with the different excitations of the IQL. The hole can recombine via annihilation of *n* QEs and creation of $(3-n)$ QHs within the conden-

sate: hQE_n → $(3-n)QH+photon$, with $n=0, 1, 2$, or 3. In this model the energy separation between the multiple PL lines is the energy needed to create a QE-QH pair. Note that in both theoretical descriptions $\nu = 1/3$ and $\nu = 2/3$ are equivalent, in contrast to our experimental data. Without a dedicated calculation it is, however, difficult to obtain a definite identification of the transitions within the PL fine structure and we anticipate that our results will stimulate additional theoretical efforts to further unravel the optical properties of correlated electron liquids.

In summary, we have shown that the radiative recombination of 2D electrons with itinerant photoexcited holes in single HJs in the FQH regime leads to remarkable anomalies in the photoluminescence spectra. A characteristic splitting of the emission band into three lines was found at filling factor $\nu = 2/3$ and in the region $2/5 > \nu > 1/3$. The broad, lowest-energy line in the emission between $\nu = 2/5$ and ν = 1/3 is tentatively attributed to indirect magnetorotonassisted transitions from the ground state of the photoexcited FOH system.¹³

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